2005a Introduction to Shimura varieties (Toronto long version)

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Erratum

Some of these have been fixed in the version under "Expository Notes".

p9, top. It is an *isomorphism* of riemannian manifolds that is called an isometry, not a morphism.

p12, footnote 10. It was H. Cartan, not E. Cartan, who proved that the group of isometries of a bounded domain has a natural structure of a Lie group; then E. Cartan proved that the group of isometries of a symmetric bounded domain is semisimple (see Borel, Essays ..., 2001, IV 6).

p30, proof of 2.14, just above the diagram 26. I write $G \xrightarrow{\mathrm{Ad}} \mathfrak{g}$ where I should write $G \xrightarrow{\mathrm{Ad}} \mathrm{GL}(\mathfrak{g})$ (from Bin Du).

p.51, l.5 ... depend *on* a (from Timo Keller).

p.59, Lemma 5.22 This is misstated: in general, $T(\mathbb{Q})$ is not closed in $T(\mathbb{A}_f)$ (unless (G, X) satisfies SV5) and so $T(\mathbb{Q})\backslash T(\mathbb{A}_f)$ is not Hausdorff (hence not compact). The last step of the proof "An arbitrary torus ..." fails when $T(\mathbb{Q})\backslash T(\mathbb{A}_f)$ is not compact. The proof of the finiteness of $T(\mathbb{Q})\backslash T(\mathbb{A}_f)/\nu(K)$ needs to be rewritten. (Bas Edixhoven)

p.67. In the display under SV1, interchange $z/\overline{z}$ and $\overline{z}/z$.

p80, 81. Lucio Guerberoff points out that the uniqueness assertion in Proposition 8.14 fails and that the condition (**) in Theorem 8.17 is inadequate. He writes (slightly edited):

> In Theorem 8.17, you say that your condition (**) on the isomorphism a is enough to guarantee that ah_A belongs to X (h_A being the morphism defining the Hodge structure on $H_1(A, \mathbb{Q})$). However I believe that this only implies that ah_A belongs to the Siegel double space of (V, ψ), but not necessarily to the $G(\mathbb{R})$-conjugacy class X. More precisely, I'm not sure if I'm missing something in Proposition 8.14. I tried to reproduce all the relevant calculations, comparing with Kottwitz's JAMS paper, and my conclusion is that if x is a morphism from $\mathbb{S}$ to $G_{\mathbb{R}}$, then x belongs to X if and only if two conditions hold:
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> 1) x lies in the Siegel double space of (V, ψ), and
>
> 2) (fix one morphism h in X) the two $B \otimes_{\mathbb{Q}} \mathbb{C}$ structures on $V \otimes_{\mathbb{Q}} \mathbb{R}$ (one is $x(i)$, other one is $h(i)$) are isomorphic.
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> Condition 1) only guarantees that they will be $\mathbb{C}$-isomorphic, not necessarily in a B-linear way. I don't see how this would be automatically implied from 1). In other words, your statement of Proposition 8.14 suggests that if x and h have target $G_{\mathbb{R}}$, satisfy 1) and are conjugate under $\mathrm{GSp}(\psi)(\mathbb{R})$, then they are also $G(\mathbb{R})$-conjugate, which doesn't seem to be the case. (For example, take a unitary group of signature (r, s) over a CM extension $K/\mathbb{Q}$, K quadratic imaginary say, and starting with the usual $h(z) = \mathrm{diag}(zI_r, \overline{z}I_s)$, consider $h'(z) = h(\overline{z})$; then h' has target $G_{\mathbb{R}}$, and is obviously on the Siegel double space (to form ψ, use a trace zero element in K), but it's not $G(\mathbb{R})$-conjugate to h unless $r = s$).
>
> In the same vein, I'm seeing condition (**) as missing something. In the same example, suppose the hermitian space defining the unitary group is $(V, \langle , \rangle)$,

and consider $(V, -\langle,\rangle)$, so it has signature (s,r). Neither of the conditions on Theorem 8.17 care about whether you look at $\langle,\rangle$ or $-\langle,\rangle$, but the Shimura variety of $-\langle,\rangle$ should be the complex conjugate of the variety for $\langle,\rangle$.

p100, top line (proof of 11.2). Delete "therefore" from "The map therefore factors through..." — as Brian Conrad reminded me the group $\mathbb{A}_{E,f}^{\times}/E^{\times}$ need not be Hausdorff. In a detailed proof, one replaces $\mathbb{A}_{E,f}^{\times}/E^{\times}$ with a quotient $T(\mathbb{A}_f)/T(\mathbb{Q})$, which is Hausdorff. Here T is a certain subtorus of $(\mathbb{G}_m)_{E/\mathbb{Q}}$. See my notes on Complex Multiplication for the details.

p124, top. Shenghao Sun has pointed out to me that the statement that pro-tori correspond to *free* $\mathbb{Z}$-modules with a continuous action of Γ is contradicted on the next page where I show that the character group of $\mathbb{G}$ is $\mathbb{Q}$. The "free" should be "torsion-free".