2001a The Tate conjecture for certain abelian varieties over finite fields

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Proof of Theorem B.1. The map $x \mapsto x_{I,J}$ sends algebraic classes to algebraic classes because it is defined by an idempotent of the ring of correspondences over Ω generated by E acting on $H^1(A)$, not an idempotent of $E \otimes \Omega$.

In fact, the proof can be simplified and clarified by using the theory of Lefschetz classes. The correspondence $x \mapsto x_{I,J}$ on $A \times A$ is Lefschetz, as is the operator Λ (so there is no need to appeal to Lieberman) — see §5 of my paper ***Lefschetz classes on abelian varieties***. Note that $L(A)$ is contained in the torus with $\mathbb{Q}$-points $\{a \in E^\times \mid a \cdot \iota a \in \mathbb{Q}^\times\}$.

For the simplest proof of Clozel's theorem (simplification of an argument of Deligne) see Milne 2009b, 1.14.