1983a The action of an automorphism of $\mathbb{C}$ on a Shimura variety and its special points (Shafarevich volume.)

(Arithmetic and geometry, Vol. I, 239–265, Progr. Math., 35, Birkhäuser Boston, Boston, MA, 1983.)

In 1981 it was generally considered that the problem of proving the existence of canonical models for the Shimura varieties not treated in Deligne's Corvallis article (that is, for those not of abelian type) was beyond reach. This perception changed with Borovoi's manuscript "Canonical models of Shimura varieties, 26.5.81". There he uses the idea (which he credits to Piatetski-Shapiro) of embedding the Shimura variety in a larger Shimura variety that contains many Shimura subvarieties of type A_1 to attempt to prove the existence of canonical models. He doesn't succeed in his attempt, and in fact no one has succeeded in making this idea work directly except for the uninteresting case of Shimura varieties defined by adjoint groups, but the idea is a fruitful one. I used it to extend Shih's and my proof of Langlands's conjugacy conjecture, which has the existence of canonical models as a consequence, to Shimura varieties not of abelian type. I talked on this at a conference at the University of British Columbia in August 1981 (manuscript 26/11/81). At the time, my proof wasn't completely general — in some cases I needed to assume that the congruence subgroup problem had a positive solution for the group. A suggestion of Deligne in early 1982 (as I recall, that a connected Shimura variety behaves as though it were simply connected) allowed me to remove the restriction, and I completed my proof (and manuscript) in early 1982. After talking to Deligne, I wrote to Borovoi (through Zarhin) offering him joint authorship. He declined, and instead announced his own proof (The Langlands conjecture on the conjugation of Shimura varieties, Functional Anal. Appl., 16 (1982), pp. 292–294).

I submitted my paper to the Shafarevich volume in April 1982, and it was published in 1983.

In May 1983, about 12 months after he had received my manuscript, Borovoi submitted his own manuscript, which was published in 1984 (Langlands' conjecture concerning conjugation of connected Shimura varieties, Sel. Math. Sov., 3 (1983/84), pp. 3–39). However, his proof in that manuscript is incomplete since it relies on a statement "Theorem 3.21" which "will be proved in another paper by the author". As far as I know, "Theorem 3.21" is still unproven[1] but Borovoi proved a weaker statement sufficient for the application to Shimura varieties in October 1986 (On the groups of points of a semisimple group over a totally real field, Problems in Group Theory and Homological Algebra, Yaroslavl 1987, pp. 142–149).

Erratum

p239. In the last displayed map, omit the second copy of $G'_{\mathbb{A}^f}$.

p256. First line: $(T,h) \overset{i}{\hookrightarrow} (G,X^+)$.

Nonerratum

p262. Theorem 7.2 asserts that canonical models exist for all Shimura varieties as a consequence of the proof of Langlands's conjugation conjecture. The original source for this

[1] In a letter to me dated July 3, 1988, Borovoi said that he thought it was his "duty to prove it".

implication is Langlands's Corvallis article, pp 233–234, which sketches the derivation of a descent datum from the conjugation conjecture and says that "one applies the Weil criterion for descent of the field of definition". In Milne and Shih 1982, Conjugates..., the sketch is made more detailed. In neither reference is it checked that the descent datum satisfies the continuity condition necessary before one can apply Weil's criterion, although there is no reason to think that Langlands did not in fact check the continuity. The fact that the continuity was not explicitly checked anywhere in the literature has achieved a certain notoriety with authors stating "this argument is flawed" and referring to a "gap in the argument" and to the need to "correct the proof". One author went so far as to claim as a "new result" the statement that had been proved 17 years earlier. In fact the descent datum does satisfy the continuity condition, and it is relatively easy to verify this. Moreover, the proof requires nothing that was not available in 1977 when Langlands wrote his article. See my article in the Michigan Math. J. 1999.