# 1976b Flat homology

(Bull. Amer. Math. Soc. 82 (1976), 118-120.).

For a scheme $X$ over a field $k$, this article proves the existence of a flat homology complex that universally computes the flat cohomology of any constant commutative algebraic group over $X$. This partially confirms, and partially contradicts, a conjecture of Grothendieck (p. 316 of Grothendieck, A. Crystals and the de Rham cohomology of schemes. 1968 Dix Exposés sur la Cohomologie des Schémas pp. 306–358 North-Holland, Amsterdam; Masson, Paris). There were so many misprints in the statement of the conjecture that it was difficult to discern what it said, so I asked Grothendieck. He responded:

> Thanks for your letter. Unfortuntately, I have no copy left of my exposé on crystals and could not tell you what the correct reading would be. Anyhow, by the time I pondered first about these things, when the exposé was written, I rather suspected that the $\mathrm{Ext}^i$ were the same in the categories you call $C_\nu$ and $C^\nu$ respectively — so I guess if I made a choice, it was at random, or rather, there was probably a preference for the fppf $\mathrm{Ext}^i$ ....

See the next page.

Dear Mike,

Thanks for your letter. Unfortunately I have no copy left on my exposé on crystals and could not tell you what the correct reading would be. Anyhow, by the time I pondered first about these things, when the exposé was written, I rather suspected that the Ext^i where the same in the categories you call $C_p$ and $C^p$ respectively — so I guess if I made a choice, it was at random; ~~[illegible]~~, or rather, there was probably a preference for fppf Ext's, as they fitted into a simpler and more general context. Is it true that you can prove the theorems with the Ext's in Pro $C_p$; can't you take $C^p$ instead ~~[illegible]~~ (X being proper over k)? To finish settling the matter, it would be nice to give an example where there does not exist a complex of sheaves in $C^p$ (or pro-$C^p$) (pro-)representing $H^i_{fppf}(X, -)$. What do you think about that? I guess there is a pb only if p = car k?

What you tell me about relation to $H^i_{cris}(X/W)$.. sounds indeed familiar to me. I had discussed the matter a few years ago with Barry Mazur, who (if I remember correctly) even tried to prove the statement you suggest. Did you discuss the matter with him?

Although I more or less stopped doing research, I still would be grateful if you keep me informed of any advances you may make, and keep me in part me / [illegible] work.

Best wishes

Alexander Grothendieck

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